Brane-world black holes and the scale of gravity

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A particle in four dimensions should behave like a classical black hole if the horizon radius is larger than the Compton wavelength or, equivalently, if its degeneracy (measured by entropy in units of the Planck scale) is large. For spherically symmetric black holes in $4+d$ dimensions, both arguments again lead to a mass threshold $M_{\rm C}$ and degeneracy scale $M_{\rm deg}$ of the order of the fundamental scale of gravity $M_{\rm G}$. In the brane-world, deviations from the Schwarzschild metric induced by bulk effects alter the horizon radius and effective four-dimensional Euclidean action in such a way that $M_{\rm C}\simeq M_{\rm deg}$ might be either larger or smaller than $M_{\rm G}$. This opens up the possibility that black holes exist with a mass smaller than $M_{\rm G}$ and might be produced at the LHC even if $M_{\rm G}\gtrsim 10\,$TeV, whereas effects due to bulk graviton exchanges remain undetectable because suppressed by inverse powers of $M_{\rm G}$. Conversely, even if black holes are not found at the LHC, it is still possible that $M_{\rm C}\gg M_{\rm G}$ and $M_{\rm G}\simeq 1\,$TeV.

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Introduction A most exciting feature of string-inspired models with large extra dimensions [1, 2] is that the fundamental scale of gravity $M_{\rm G}$ could be much smaller than the Planck mass $M_{\rm P}\simeq 10^{16}\,$TeV and as low as the electro-weak scale ($M_{\rm G}\simeq 1\,$TeV). Microscopic black holes may therefore be created in our accelerators [3–5] with a production cross section given, according to the hoop conjecture [6], by $\sigma\sim R_{\rm H}^2$ [4], where $R_{\rm H}$ is the radius of the forming horizon and is bounded below by the wavelength of typical quantum fluctuations [7, 8]. After the black hole has formed (and possible transients), the Hawking radiation [9] is expected to set off, with the most common description based on the canonical Planckian distribution for the emitted particles and consequent instantaneous decay [4]. This standard picture and a variety of refinements have been implemented in the most recent Monte Carlo codes [10] and the outcome is being confronted with Large Hadron Collider (LHC) data [11, 12]. One problem with the canonical description is that the black hole specific heat is in general negative and one should therefore use the more consistent microcanonical description [13, 14], which however requires an explicit counting of the black hole microscopic degrees of freedom (or degeneracy).

For this counting, one may appeal to the *area law* [15], from which it can be inferred the horizon area describes the black hole degeneracy [16, 17]. The area-entropy correspondence has inspired the holographic principle [18] in order to solve the black hole information paradox [19]. This principle has widely been developed [20], and a theoretical support was found in the AdS/CFT correspondence [21] that conjectures the equivalence of a string theory with gravity in anti-de Sitter space with a quantum field theory without gravity on the boundary. In this letter we shall analyze the interplay between the classicality condition that must be met in black hole formation and the horizon area as a measure of the entropy of black holes in the brane-world [2]. Our results allow for the existence of "lightweight" microscopic black holes (LBH) with mass below $M_{\rm G}$, or could explain the non existence of black holes within the reach of LHC experiments [11, 12] even if $M_{\rm G}\simeq 1\,$TeV.

Compton classicality A black hole is a classical space-time configuration and its production in a collider is therefore a "classicalization" process in which quantum mechanical particles are trapped by gravitational self-interaction within the horizon [7, 8]. Consequently, quantum fluctuations should be negligible for the final state (of total energy M) which sources such a metric. A widely accepted condition of classicality is then expressed by assuming the Compton wavelength $\lambda_{\rm C}\simeq\hbar/M=\ell_{\rm P}\,M_{\rm P}/M$ [1] of the black hole, viewed as *one particle*, is the lower bound for the "would-be horizon radius" $R_{\rm H}$, that is

$$R_{\rm H}\gtrsim\lambda_{\rm C}\ , \tag{1}$$

where $R_{\rm H}=R_{\rm H}(M)$ depends on the specific black hole metric. In four dimensions, using the Schwarzschild metric, one obtains

$$R_{\rm H}=2\,\ell_{\rm P}\,\frac{M}{M_{\rm P}}\gtrsim\ell_{\rm P}\,\frac{M_{\rm P}}{M}\quad\Rightarrow\quad M\gtrsim M_{\rm C}\simeq M_{\rm P}\ , \tag{2}$$

which is supported by perturbative calculations of scattering amplitudes for particles with centre-mass energy M [8]. The above derivation does not make full use of the space-time geometry and, in particular, neglects that, for M approaching the scale $M_{\rm G}$, quantum fields should be affected by extra-spatial dimensions (if they exist).

[1] We shall mostly use units with the Boltzmann constant $k_{\rm B}=c=1$, $G_{\rm N}=\ell_{\rm P}/M_{\rm P}$ and $\hbar=\ell_{\rm P}\,M_{\rm P}=\ell_{\rm G}\,M_{\rm G}$.

Entropic classicality Another classicality argument can be given, which does not involve black hole wavefunctions but relies on Bekenstein's conjectured correspondence between the entropy of thermodynamical systems and the area of black hole horizons [16]. Christoudolou [22] first pointed out that the irreducible mass $M_{\rm ir}$ of a Kerr black hole, *i.e.* the amount of energy that cannot be converted into work by means of the Penrose process [23], is related to the horizon area A as $M_{\rm ir} = M_{\rm P}\,\sqrt{A/16\,\pi\,\ell_{\rm P}^2} \equiv \sqrt{A_{\rm ir}}$. Now, in thermodynamics, an increase in entropy is associated with a degradation of energy because the work we can extract from the system is reduced. The similarity is clear, but goes beyond this simple statement. For a Schwarzschild black hole, $M_{\rm ir} = M$ and no energy at all can be extracted. Nonetheless, we can take a collection of fully degraded subsystems (Schwarzschild black holes) and still get some work out of them. In fact, if we merge two or more black holes, the total horizon area must equal at least the sum of all their original areas [15]. Denoting by M_i and A_i the initial irreducible masses and areas, and by M_F and A_F the final irreducible mass and horizon area, we see that $M_F = \sqrt{A_F} = \sqrt{\sum_i A_i} < \sum_i \sqrt{A_i} = \sum_i M_i$. The final irreducible mass is then less than the sum of all the initial irreducible masses: some more work can be extracted by merging fully degraded black holes. The same occurs by collecting thermodynamical systems that – individually – are fully degraded but together can still provide work. Bekenstein [16] remarked how we can clarify these similarities by invoking Shannon's entropy

$$S = -\sum_n p_n\,\ln p_n\ , \tag{3}$$

where p_n is the probability for a thermodynamical system to be found in the n-th state. A thermodynamical system is described in terms of a few macroscopical variables (like energy, temperature and pressure). Once these variables are fixed, the system can however be described by a huge amount of *microscopically inequivalent* states. Hence, entropy can be seen as the lack of information about the actual internal structure of the system. Analogously, any four-dimensional black hole can be described in terms of three macroscopic variables: mass, angular momentum and charge. All information about the matter which formed the black hole is lost beyond the horizon. Because of properties shared by thermodynamical entropy and horizon area, Bekenstein found the simplest expression (with dimensions of $\hbar$) which satisfies the conditions on the irreducible mass is

$$S_{\rm BH} = \frac{M_{\rm P}\,A}{16\,\pi\,\ell_{\rm P}}\ . \tag{4}$$

Using a *gedanken experiment*, Bekenstein [24] further obtained the so-called entropy bound $S_{\rm BH} \le 2\,\pi\,R_{\rm H}\,M$, and this topic has by now been extended to more general scenarios (for a review, see Ref. [25]).

From Eq. (4), we can now infer an *entropic condition* for black hole classicality: a four-dimensional classical black hole should have a large degeneracy (in units of the Planck scale), that is

$$\tilde{S}^{\rm E}_{(4)} \equiv \frac{S_{\rm BH}}{\ell_{\rm P}\,M_{\rm P}} = \frac{4\,\pi\,R_{\rm H}^2}{16\,\pi\,\ell_{\rm P}^2} \simeq \left(\frac{M}{M_{\rm deg}}\right)^2 \gtrsim 1\ , \tag{5}$$

which, for the Schwarzschild metric, leads to $M \gtrsim M_{\rm deg} \simeq M_{\rm P}$. This conclusion is also supported by perturbative calculations of scattering amplitudes, since the entropy (4) can be reproduced by assuming the final classical black holes are composed of quanta with wavelength $\lambda \sim R_{\rm H}$ [8]. Note, however, that the physical meaning of the two scales is not quite the same: $M_{\rm deg}$ is the natural unit for measuring black hole internal degrees of freedom (like the gap between energy levels of the harmonic oscillator), whereas $M_{\rm C}$ is the minimum mass below which black holes do not exist (like the threshold in massive particle production). That $M_{\rm deg} \simeq M_{\rm C} \simeq M_{\rm P}$ is expected – because gravity in four dimensions entails one scale – but is till a remarkable evidence that black holes hide most information about forming matter.

ADD black holes Both classicality conditions (2) and (5) can be straightforwardly generalized to models with extra-spatial dimensions by replacing $M_{\rm P}$ and $\ell_{\rm P}$ with $M_{\rm G}$ and $\ell_{\rm G}$, and using the appropriate expressions for the horizon radius. For example, in the ADD scenario of Refs. [1], the brane tension is neglected and one can therefore consider vacuum solutions to the Einstein equations in $4+d$ dimensions to derive the following relation between the mass and horizon radius [4],

$$R_{\rm H} = \frac{\ell_{\rm G}}{\sqrt{\pi}}\left(\frac{M}{M_{\rm G}}\right)^{\frac{1}{1+d}}\left(\frac{8\,\Gamma\left(\frac{d+3}{2}\right)}{2+d}\right)^{\frac{1}{1+d}}\ , \tag{6}$$

where Γ is the usual Gamma function. Inserting the above into Eq. (1) yields [2]

$$R_{\rm H} \gtrsim \ell_{\rm G}\,\frac{M_{\rm G}}{M} \qquad \Rightarrow \qquad M \gtrsim M_{\rm C} \simeq M_{\rm G}\ , \tag{7}$$

as one would naively expect. Moreover, the same result is again obtained by generalizing the entropic argument to $4+d$ dimensions, namely

$$\tilde{S}^{\rm E}_{(4+d)} \simeq \left(\frac{R_{\rm H}}{\ell_{\rm G}}\right)^{2+d} \sim \left(\frac{M}{M_{\rm deg}}\right)^{\frac{2+d}{1+d}} \gtrsim 1\ , \tag{8}$$

where $M_{\rm deg} \simeq M_{\rm G}$. One therefore concludes that even in the ADD scenario, gravity enters black hole physics with one scale, $M_{\rm G}$, like in four dimensions.

[2] This is the kind of condition employed in all Monte Carlo studies of black hole production at the LHC [10].

Brane-world black holes The situation appears more involved in the brane-world (RS) scenario [2], in which the brane tension is not ignored and the bulk is consequently warped. This has made it very hard to describe black holes [26] [3], and only a few analytical candidates are known which solve the effective four-dimensional vacuum Einstein equations [28],

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \mathcal{E}_{\mu\nu} \qquad \Rightarrow \qquad R = 0 \ , \tag{9}$$

where the presence of tidal effects from the propagation of gravity into the bulk is represented by the (traceless) projected Weyl tensor $\mathcal{E}_{\mu\nu}$. One of these solutions is the tidally charged metric [29]

$$ds^2 = -A\, dt^2 + A^{-1}\, dr^2 + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \ , \tag{10}$$

with

$$A = 1 - \frac{2\,\ell_{\rm P}\, M}{M_{\rm P}\, r} - q\,\frac{\ell_{\rm G}^2}{r^2} \ , \tag{11}$$

which has been extensively studied in Refs. [14, 30–32]. In the above and what follows, the tidal charge q and M are treated as independent quantities, although one expects q vanishes when the black hole mass $M = 0$. Further, we only consider the case $q > 0$ since negative tidal charge would yield anti-gravity effects [31]. A relation $q = q(M)$ should be obtained by solving the complete five-dimensional Einstein equations [32] (or by means of supplementary arguments [33]). Nevertheless, since we are here interested in black holes near their minimum possible mass $M_{\rm C} \sim M_{\rm G} \ll M_{\rm P}$, we can approximate $q \simeq q(M_{\rm C})$ and constant, and expand all final expressions for $M \sim M_{\rm G} \ll M_{\rm P}$.

We can first apply the usual classicality argument (1), with the horizon radius

$$R_{\rm H} = \ell_{\rm P} \left(\frac{M}{M_{\rm P}} + \sqrt{\frac{M^2}{M_{\rm P}^2} + q\,\frac{M_{\rm P}^2}{M_{\rm G}^2}} \right) \ , \tag{12}$$

and obtain $M \gtrsim M_{\rm C}$, where the minimum mass

$$M_{\rm C} \simeq \frac{M_{\rm G}}{\sqrt{q}} \ , \tag{13}$$

for $M \sim M_{\rm G} \ll M_{\rm P}$.

We can also repeat the entropic argument by employing the effective four-dimensional action, namely

$$\tilde{S}^{\rm E}_{\rm eff} \simeq \frac{4\,\pi\, R_{\rm H}^2}{16\,\pi\,\ell_{\rm P}^2} \ , \tag{14}$$

which, however, is not at a minimum for $M \simeq M_{\rm C}$, as one would instead expect from previous cases. This discrepancy can be cured by recalling the Euclidean action (as well as the thermodynamical entropy) is defined modulo constant terms, which, for example, do not affect the value of the Hawking temperature nor the microcanonical description of the Hawking radiation [13]. By subtracting from Eq. (14) a suitable constant, namely

$$\tilde{S}^{\rm E}_{\rm sub} = \tilde{S}^{\rm E}_{\rm eff}(M) - \tilde{S}^{\rm E}_{\rm eff}(M_{\rm C}) \ , \tag{15}$$

and expanding for $M \sim M_{\rm G} \ll M_{\rm P}$, we finally obtain

$$\tilde{S}^{\rm E}_{\rm sub} \simeq \frac{M}{M_{\rm deg}} \ , \tag{16}$$

where there now appears the effective degeneracy scale

$$M_{\rm deg} \simeq \frac{M_{\rm G}}{\sqrt{q}} \ . \tag{17}$$

It is again remarkable that $M_{\rm deg} \simeq M_{\rm C}$ and brane-world black holes are also described by one scale [recall that $q \simeq q(M_{\rm C})$ is not truly independent]. The "natural" choice would now be $q \simeq 1$, so that $M_{\rm deg} \simeq M_{\rm C} \simeq M_{\rm G}$, but the effective scale $M_{\rm deg} \simeq M_{\rm C}$ could also be either larger ($q \ll 1$) or smaller ($q \gg 1$) than $M_{\rm G}$.

Concluding remarks Detection of black holes at the LHC would be a clear signal that we are embedded in a higher-dimensional space-time and the fundamental scale of gravity $M_{\rm G} \simeq 1\,$TeV. The existence of extra spatial dimensions could also be uncovered by means of particle processes which involve the exchange of bulk gravitons [35]. Such processes are perturbatively described by operators suppressed by inverse powers of $M_{\rm G}$, and might not be detectable if the latter is larger than a few TeV [36]. We have shown that both classicality conditions, from quantum mechanics and the entropic counting of internal degrees of freedom, allow for brane-world black holes with minimum mass (13). The latter might be different from the fundamental scale $M_{\rm G}$, if q departs significantly from 1 (which should be related to details of the mechanism confining standard model particles and four-dimensional modes of gravity on the brane). This introduces two alternative scenarios:

i) for $q \gg 1$, "lightweight black holes" (LBH) with $M_{\rm C} \lesssim M \lesssim M_{\rm G}$ may exist and be produced at the LHC even if $M_{\rm G} \gtrsim 10\,$TeV. In this case, the effects due to bulk graviton exchanges would remain undetected;

ii) if $q \ll 1$, black holes do not exist with $M \simeq M_{\rm G}$, even if $M_{\rm G} \simeq 1\,$TeV, and processes involving bulk gravitons are the only available signature of extra-spatial dimensions.

The former scenario might have important phenomenological implications both for accelerator physics and in astrophysics. Although recent LHC data at 7 TeV center mass energy seem to exclude the production of

[3] Arguments have been formulated against the existence of static brane-world black hole metrics [27]. Given the Hawking radiation is likely a strong effect for microscopic black holes, their instability is here taken as granted.

microscopic black holes [11, 12], there is still the possibility that future runs at 14 TeV will achieve this goal. Further, LHB might play a role in cosmological models as primordial black holes produced in the early universe, and in astrophysics as the outcome of high energy cosmic rays colliding against dense stars [37].

The case of $q \ll 1$ might instead explain why there is no evidence of black holes at the LHC [11, 12], even if extra-spatial dimensions exist.

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